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AN ANALYTICAL AND EXPERIMENTAL INVESTIGATION OF THE NATURAL FREQUENCIES OF UNIFORM RECTANGULAR-CROSS-SECTION FREE-FREE SANDWICH BEAMS

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SUMMARY

Measurements were made of the natural frequencies corresponding to the first six modes of three uniform rectangular-cross-section sandwich beams supported to simulate free-free conditions. Data were also obtained concerning the change of the natural frequencies corresponding to the first five modes of one of the three beams when the density of the air surrounding the beam was varied.

Comparison of the measured natural frequencies with those predicted by the classical theory of beam vibrations indicated large differences, particularly for the higher modes. Correlation of the measured natural frequencies with those predicted by the Timoshenko beam theory, which includes the effect of transverse shear and rotary inertia, was excellent for all six modes. For all beams tested, the effect of rotary inertia was found to be negligible. The natural frequencies of a lightweight sandwich beam were found to increase as the density of the air surrounding the vibrating beam was decreased.

INTRODUCTION

Lightweight structures which have high strength and stiffness properties are meeting the requirements in many structural design problems. Therefore, sandwich constructions, which are characteristically lightweight and comparatively rigid, are being given increasing consideration for varied applications such as in the primary structure for ballistic missiles and space vehicles, secondary structures for airplanes, wall panels for houses, and furniture. In general, a sandwich structure consists of a low-density core bonded between two relatively thin face plates as shown in figure 1.

Most of the literature pertaining to sandwich structures treats theoretical and experimental investigations with static loads (see, e.g., refs. 1, 2, and 3). Only limited literature is available concerning either theoretical analysis of dynamic loads on sandwich structures (see, e.g., refs. 4, 5, and 6) or experimental results pertaining to the natural frequencies and mode shapes of vibrating sandwich structures (see, e.g., refs. 7 and 8).

The application of the Timoshenko beam theory (ref. 9) for sandwich beams is discussed and used in reference 7 to predict the frequencies of the first five lateral modes of sandwich beams with various end supports. Average values of the flexural rigidity and shear stiffness of each beam were calculated from the experimental frequencies obtained with both ends simply supported. These values were used with the frequency equations developed from the Timoshenko beam theory to predict the natural frequencies of the beams with other end supports. The correlation between the experimental and theoretical frequencies was not good and no explanation was given for the difference.

Reference 8 presents a method which can be used to predict the natural frequency of the first lateral mode of sandwich beams. In this reference, expressions for the kinetic and potential energies of the beam, including shear effects, are derived, and the natural frequency is obtained by equating the maximum kinetic energy to the maximum potential energy. The method becomes less accurate and unduly laborious for natural frequencies corresponding to the higher modes.

The purposes of the present investigation were to derive a simple method for predicting the natural frequencies of a free-free sandwich beam, and to evaluate the variation of natural frequency with core thickness, face thickness, beam length, and density of the surrounding air.

SYMBOLS

A cross-sectional area, in.^{2}

a_n frequency coefficient used in equation given by classical beam theory where secondary effects are neglected

b width of beam, in.

d distance between centroidal axis of beam and centroidal axis of one of its faces, in.

E Young's modulus of elasticity, psi

EI flexural stiffness, lb-in.^{2}

f experimental natural frequency of beam unless otherwise noted, cps

G core shear modulus, psi

I total centroidal area moment of inertia of cross section, in.^{4}

I_0 centroidal area moment of inertia, in.^{4}

k_B frequency coefficient used in equations for determining transverse-shear and rotary-inertia effects

k_{RI} rotary-inertia coefficient

k_S shear-rigidity coefficient

L length, in.

t thickness, in.

W weight, lb

α, β constants used in equations for determining transverse-shear and rotary-inertia effects

μ mass per unit length of beam, lb-sec^2/in.^{2}

Subscripts:

as air shaker

b beam

c core of beam

e effective

es electromagnetic shaker

f face of beam

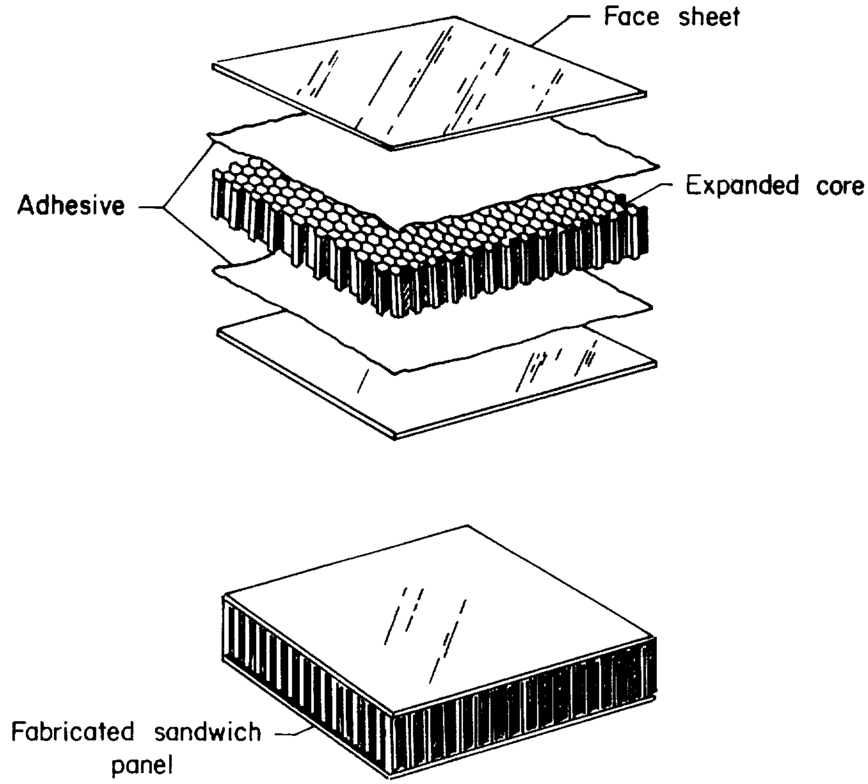
n mode number, $n = 1, 2, \ldots, 6$

th theoretical neglecting secondary effects

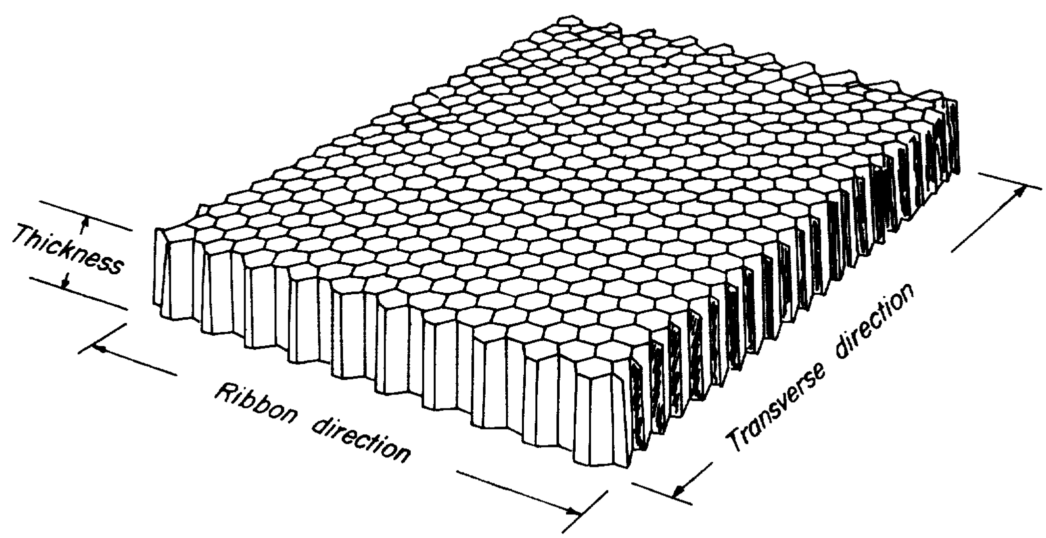
th,s theoretical including secondary effects

APPARATUS AND TEST PROCEDURE

Three sandwich beams, constructed of aluminum faces and orthotropic aluminum honeycomb cores, were used in this investigation. (See fig. 1.) The cores were constructed of 0.001-inch-thick permeated 3003-H19 aluminum alloy. The cell size was 3/16 inch and the cores were oriented between the faces so that the transverse direction of the honeycomb was in the direction of the length of the beam. Dimensions and properties of the core, faces, and beams are given in tables I, II, and III.



(a) Primary parts.



(b) Core notation.

Figure 1.- Sandwich structure.

TABLE I.- CORE DIMENSIONS AND PROPERTIES

Specimen	Thickness, in.	Width, in.	Length, in.	Weight, lb	Transverse shear modulus, psi	Natural frequency, cps	Flexural stiffness, lb-in.^{2}
Beam 1	2	6	36	0.77	20,000	4.0	119
Beam 2	2	6	36	.89	20,000	3.8	122
Beam 3	8	16	96	22.06	20,000	2.9	33,500

TABLE II.- FACE DIMENSIONS AND PROPERTIES

Specimen	Thickness, in.	Width, in.	Length, in.	Weight, lb	Modulus of elasticity, psi
Beam 1	0.016	6	36	0.36	10×10^6
Beam 2	.064	6	36	1.40	10
Beam 3	.064	16	96	10.05	10

TABLE III.- BEAM DIMENSIONS AND PROPERTIES

Specimen	Thickness, in.	Width, in.	Length, in.	Weight, lb	Flexural stiffness, lb-in.^{2}
Beam 1	2.032	6	36	1.65	1.95×10^6
Beam 2	2.128	6	36	3.77	8.18
Beam 3	8.128	16	96	43.49	333

After each core was cut to the proper dimensions and weighed, the core alone was suspended in a free-free condition (see fig. 2) and an air shaker (ref. 10) was used to obtain the natural frequency of its first mode. Each core was very flexible and thus the resonant frequency could be obtained by varying the shaker frequency until maximum displacement of the core was visually observed. These data were used to calculate the flexural stiffness of the core.

The cores of beams 1 and 2 were bonded to their faces with an adhesive composed of $23\frac{1}{2}$ parts of a polysulfide elastomer to 100 parts of a filled epoxy resin. The core of beam 3 was bonded to its faces with a structural adhesive film. The beams were then weighed. Next, they were suspended in a free-free condition and vibrated to obtain their natural frequencies. The natural frequencies given are those frequencies corresponding to the maximum resonance response for the modes in question. The natural frequency of the first mode for each beam was obtained with both the air shaker and an electromagnetic shaker. The natural frequencies of the higher modes of the beams were beyond the range of the air shaker; thus only the electromagnetic shaker was used for these modes. In order to ascertain conditions of resonance when using the electromagnetic shaker, a lightweight crystal accelerometer was placed near the edge of the beam and its output was amplified and applied to the vertical deflection plates of an oscilloscope. The signal from an oscillator was used to drive the electromagnetic shaker and was applied to a digital counter and to the horizontal deflection plates of the oscilloscope. The resulting pattern on the oscilloscope, normally referred to as a Lissajous figure, indicated the phase and frequency relations of the oscillator signal and the accelerometer signal. The nodal patterns of the first six modes at resonance were obtained by moving the accelerometer along the length of the beam and noting locations where the displacement was relatively small.

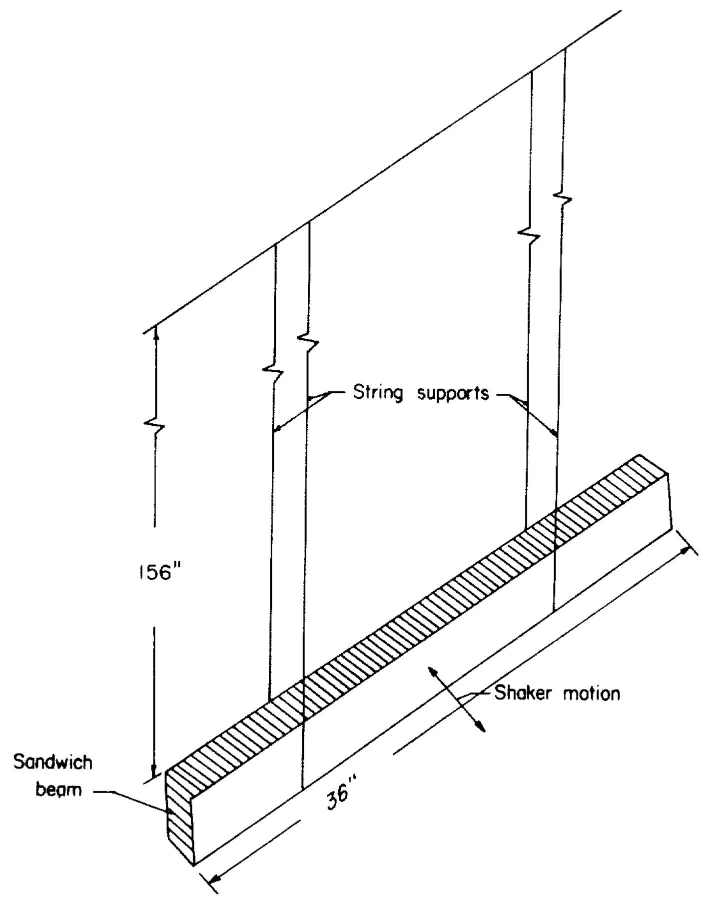


Figure 2.- Support system simulating free-free boundary conditions.

Another phase of this study was to investigate the effect of varying the density of the air surrounding a sandwich beam. Beam 2, supported to simulate free-free conditions, was placed in a large vacuum chamber and excited with the electromagnetic shaker. The vacuum chamber was the upstream section of a variable-pressure wind tunnel where the total volume is approximately 24,000 cubic feet as compared with the model volume of 1/4 cubic foot. The wires supporting the model in the free-free condition were about 6 feet long. The natural frequencies corresponding to the first five modes of the beam were recorded at pressures of 29.9, 14.0, 7.0, 3.0, and 0.24 inches of mercury, absolute. Sufficient time for stabilization of air pressure and temperature was allowed between each measurement. The aforementioned technique used to determine the natural frequencies of the beam with the electromagnetic shaker under atmospheric conditions was also used for the tests in the vacuum chamber.

RESULTS AND DISCUSSION

Analytical and Experimental Determination of the Natural Frequencies

The natural frequency of the first mode of each core when excited with the air shaker is shown in table I. Although the cores of beams 1 and 2 had the same overall dimensions, the core of beam 1 weighed less and had a higher natural frequency than the core of beam 2. The flexural stiffness of each core E_cI_c was estimated from the classical beam theory equation $E_cI_c = \frac{\left(f_{1,c}\right)^2 L_c{}^3 W_c}{4900}$, where $f_{1,c}$ is the frequency of the core obtained experimentally, L_c is the length of the core, and W_c is the weight of the core. This equation does not include effects of transverse shear or rotary inertia. The results estimated by this equation are shown in table I and show that the flexural stiffnesses of the cores of beams 1 and 2 are nearly the same.

The analytical and experimental natural frequencies of beams 1, 2, and 3 are shown in table IV. When the air shaker was used as the exciter, only the first mode of each beam could be excited accurately. However, with the use of the electromagnetic shaker and an accelerometer, the first six modes were easily excited and the corresponding natural frequencies and mode shapes could thereby be determined.

Although the flexural rigidity of beam 3 was larger than the flexural rigidities of beams 1 and 2, the natural frequencies of beam 3 were lower due to its greater length and weight. The natural frequencies of the first and second modes of beam 1 are lower than the corresponding frequencies of beam 2, but the natural frequencies of the third, fourth, fifth, and sixth modes of beam 1 are higher than the corresponding frequencies of beam 2. This trend indicates large secondary effects, such as effects of transverse shear and rotary inertia, and demonstrates that increasing the face thickness of a configuration does not necessarily result in higher natural frequencies for all modes as classical beam theory would indicate.

Table IV shows that the natural frequencies of the first mode of beams 1 and 2 were lower for tests with the electromagnetic shaker than for tests with the air shaker. These lower frequencies were due to the added mass of the

TABLE IV.- ANALYTICAL AND EXPERIMENTAL NATURAL FREQUENCIES OF BEAMS 1, 2, AND 3

Method of obtaining natural frequencies	Natural frequencies (in cps) for mode:					
	1	2	3	4	5	6
Beam 1						
Experimental:						
With air shaker	341	-----	-----	-----	-----	-----
With electromagnetic shaker	328	839	1,383	2,104	2,638	3,127
Classical beam theory:						
Based on weight of beam	352	971	1,906	3,143	4,690	6,550
Based on weight of beam and attachments	345	951	1,868	3,080	4,597	6,415
Based on effective weight of beam and attachment	339	934	1,835	3,025	4,510	6,320
Timoshenko beam theory:						
Based on weight of beam	341	855	1,468	2,169	2,861	3,603
Based on weight of beam and attachments	335	837	1,438	2,125	2,804	3,528
Based on effective weight of beam and attachments	329	822	1,413	2,087	2,751	3,476
Beam 2						
Experimental:						
With air shaker	419	-----	-----	-----	-----	-----
With electromagnetic shaker	411	887	1,340	1,825	2,236	2,674
Classical beam theory:						
Based on weight of beam	478	1,317	2,585	4,263	6,365	8,880
Based on weight of beam and attachments	473	1,304	2,561	4,223	6,304	8,798
Based on effective weight of beam and attachments	469	1,291	2,533	4,181	6,241	8,720
Timoshenko beam theory:						
Based on weight of beam	416	882	1,370	1,833	2,291	2,664
Based on weight of beam and attachments	412	874	1,357	1,816	2,269	2,639
Based on effective weight of beam and attachments	408	865	1,342	1,798	2,247	2,616
Beam 3						
Experimental:						
With air shaker	191	-----	-----	-----	-----	-----
With electromagnetic shaker	191	441	709	1,008	1,260	1,536
Classical beam theory:						
Based on weight of beam	206	568	1,116	1,840	2,747	3,833
Based on weight of beam and attachments	206	568	1,115	1,838	2,744	3,829
Based on effective weight of beam and attachments	206	568	1,116	1,840	2,747	3,833
Timoshenko beam theory:						
Based on weight of beam	190	437	714	994	1,264	1,533
Based on weight of beam and attachments	190	437	714	993	1,262	1,532
Based on effective weight of beam and attachments	190	437	714	994	1,264	1,533

apparatus required to attach the electromagnetic shaker to the beam, the mass of the moving coil of the electromagnetic shaker, and the mass of the accelerometer. The weight of each beam for the various tests is as follows:

	Beam 1	Beam 2	Beam 3
Weight of beam for tests with air shaker, lb	1.65	3.77	43.49
Weight of beam for tests with electromagnetic shaker, lb	1.72	3.84	43.56

These data indicate a 4.24-percent, 1.86-percent, and 0.16-percent increase in the weights of beams 1, 2, and 3, respectively, when the air shaker was replaced with the electromagnetic shaker. Because of the small increase in the weight of beam 3 (0.16 percent) due to the mass of the attachments, there was no measurable change in the natural frequency of the first mode of beam 3 when the air shaker was replaced with the electromagnetic shaker.

The effective weight $W_{b,e}$ of each beam when excited with the electromagnetic shaker was calculated from the equation $W_{b,e} = \left(\frac{f_{1,as}}{f_{1,es}}\right)^2 W_b$, where $f_{1,as}$ and $f_{1,es}$ are the natural frequencies corresponding to the first mode obtained experimentally with the air shaker and the electromagnetic shaker, respectively, and W_b is the weight of the beam. The calculated effective weights of beams 1, 2, and 3 are 1.78, 3.92, and 43.56 pounds, respectively.

The theoretical flexural stiffness of each beam E_bI_b was calculated with the aid of the parallel-axis theorem as follows:

$$E_bI_b = E_cI_c + 2E_fI_f = E_cI_c + 2E_f\left(I_{0,f} + A_fd^2\right)$$

where E_fI_f is the flexural stiffness of the faces, E_f is Young's modulus of elasticity of the faces, $I_{0,f}$ is the centroidal area moment of inertia of one face, A_f is the cross-sectional area of one face, and d is the distance between the centroidal axis of the beam and the centroidal axis of one of its faces. For each beam the core flexural stiffness E_cI_c was negligible (less than 0.01 percent) in comparison with the flexural stiffness due to the faces E_fI_f. The calculated flexural stiffnesses of beams 1, 2, and 3 are 1.95×10^6, 8.18×10^6, and 333×10^6, respectively.

The theoretical natural frequencies $f_{th,n}$ of the first six modes of each beam were first obtained from the equation given by the classical theory of beam vibrations $f_{th,n} = a_n\sqrt{\frac{E_bI_b}{\mu L_b^4}}$, where μ is the mass per unit length of the beam, L_b is the length of the beam, and a_n is a frequency coefficient dependent upon the number of the mode. The classical theory, which is often called the

Bernoulli-Euler theory, does not include any effects on the natural frequencies due to transverse shear and rotary inertia (ref. 11). The frequencies predicted from this theory are given in table IV for each of the following weights: (1) the actual weight of the beam, (2) the weight of the beam plus the weight of the attachments, and (3) the effective weight of the beam plus attachments. A comparison of the results obtained with these weights indicates the contribution of the shakers and the attachments.

The natural frequencies measured with the electromagnetic shaker for the first six modes of each beam are less than the frequencies calculated from the classical beam theory in which the various beam weights were used. This difference is small for all beams for the first mode, but the difference increases rapidly for the higher modes.

The natural frequencies corresponding to the first six modes of each beam, predicted by the Timoshenko beam theory (refs. 9, 11, and 12), are also given in table IV. These frequencies were calculated by the method given in reference 12 in which a theoretical analysis of the effect of transverse shear and rotary inertia on the natural frequencies of a uniform beam is presented. Curves are shown in this reference for the coefficient of shear rigidity k_S plotted against the ratio of the theoretical natural frequency including secondary effects to the theoretical natural frequency neglecting secondary effects for various values of the coefficient of rotary inertia k_{RI}. The curves were obtained from the following equations:

(a) For the symmetrically vibrating (odd-numbered modes) free-free beams

$$\beta\left(\beta^2 - k_S^2\right)\tanh k_B\alpha + \alpha\left(\alpha^2 + k_S^2\right)\tan k_B\beta = 0$$

(b) For the antisymmetrically vibrating (even-numbered modes) free-free beams

$$\alpha\left(\alpha^2 + k_S^2\right)\tanh k_B\alpha - \beta\left(\beta^2 - k_S^2\right)\tan k_B\beta = 0$$

In these equations

$$\alpha = \sqrt{\frac{-\left(k_S^2 + k_{RI}^2\right) + \sqrt{\left(k_S^2 - k_{RI}^2\right)^2 + \frac{4}{k_B^2}}}{2}}$$

$$\beta = \sqrt{\frac{\left(k_S^2 + k_{RI}^2\right) + \sqrt{\left(k_S^2 - k_{RI}^2\right)^2 + \frac{4}{k_B^2}}}{2}}$$

$$k_S = \frac{1}{L_b}\sqrt{\frac{E_b I_b}{t_c b G}}$$

$$k_B = 2\pi f_{th,s}\sqrt{\frac{\mu L_b^4}{16 E_b I_b}}$$

where $k_{RI} = \frac{1}{L_b}\sqrt{\frac{I_b}{A_b}}$, t_c is the core thickness, b is the beam width, and G is the core shear modulus in the transverse direction. The coefficient of shear rigidity k_S for each beam and the frequency correlation factor $\frac{f_{th,s}}{f_{th}}$ for the first six modes of each beam are given in table V. The rotary-inertia coefficients k_{RI} of the beams tested in this investigation were approximately 0 and were therefore neglected.

TABLE V.- TRANSVERSE SHEAR PARAMETERS

Beam	Shear-rigidity coefficient, k_S	Frequency correlation factor, $f_{th,s}/f_{th}$, for mode:					
		1	2	3	4	5	6
1	0.16	0.97	0.88	0.77	0.69	0.61	0.55
2	.32	.87	.67	.53	.43	.36	.30
3	.24	.92	.77	.64	.54	.46	.40

The theoretical frequencies corresponding to the first mode of each beam, predicted by the Timoshenko beam theory with the use of the actual weight of the beams, are in very good agreement with the experimental frequencies obtained with the air shaker. (See table IV.) The differences between the analytical and experimental frequencies are 0, 0.71, and 0.52 percent for beams 1, 2, and 3, respectively.

The experimental frequencies obtained with the electromagnetic shaker and the frequencies predicted by the Timoshenko theory were in good agreement for all modes of each beam when the weight of the beam and attachments or the effective weight of the beam and attachments was used. The maximum difference in the first mode between the analytical and experimental frequencies was 2.13 percent for beam 1 and that in the sixth mode was 13.0 percent for beam 1. The analytical and the experimental natural frequencies are plotted in figure 3. Since the only difference between the classical beam theory and the Timoshenko beam theory is the inclusion of secondary effects, such as transverse shear in the Timoshenko theory, it is evident that secondary effects must be considered when predicting the natural frequencies of sandwich beams.

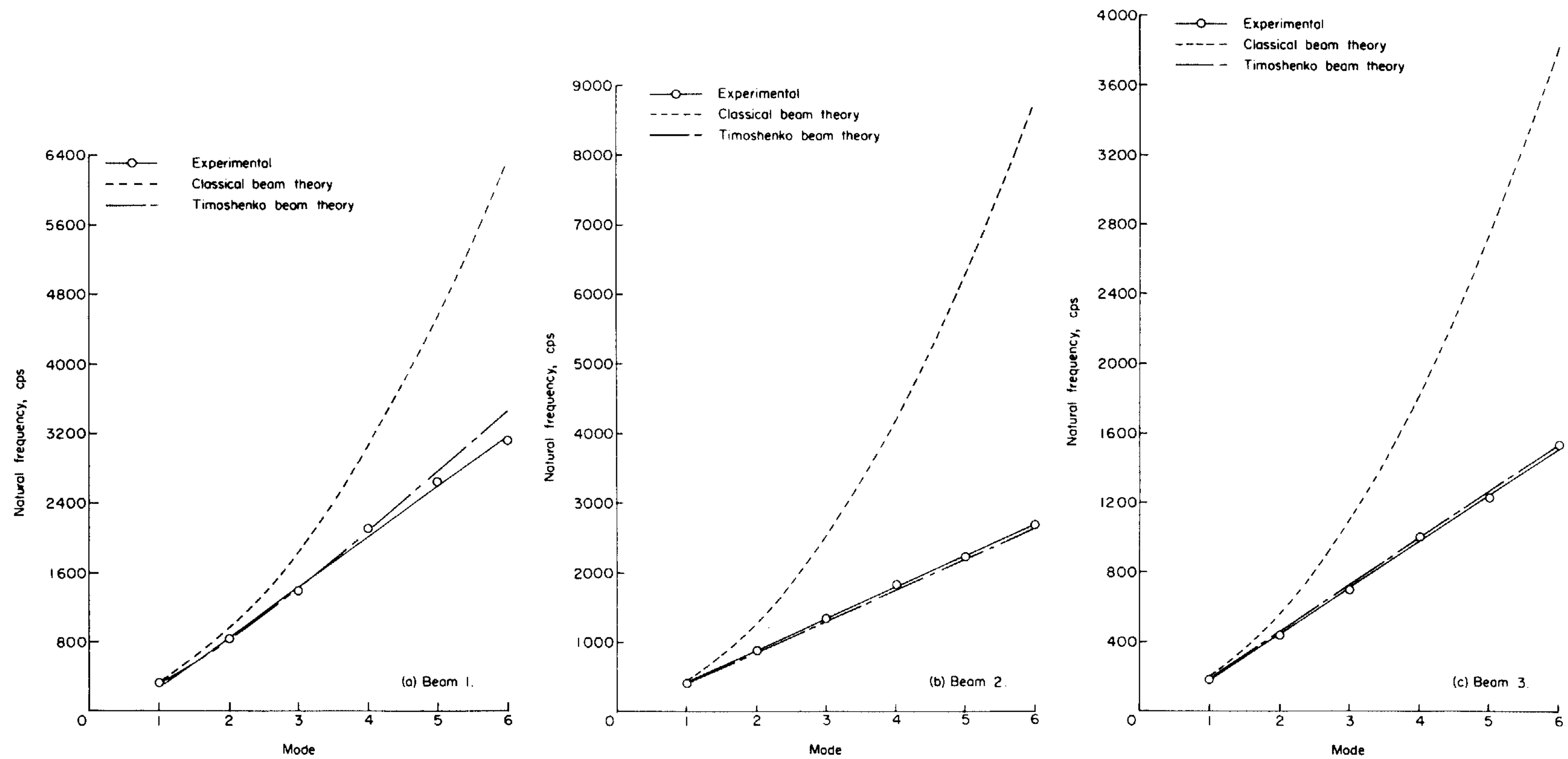


Figure 3.- Analytical and experimental natural frequencies corresponding to the first six modes.

Energy methods including transverse shear effects, such as the one presented in reference 8, have been used to predict the natural frequencies corresponding to the first mode of sandwich beams. However, the Timoshenko theory including secondary effects is more easily used for predicting the natural frequencies corresponding to higher modes and gives excellent results for the first six modes.

As previously stated in the section entitled "Apparatus and Test Procedure," a different type of adhesive was used for beam 3 than for beams 1 and 2. In each case the flexural stiffness of the thin adhesive layer was neglected and it was assumed that the distribution of the adhesive was uniform along the length of the beam and thin enough so as not to affect appreciably the shear modulus of the core. Since the natural frequencies predicted by the Timoshenko beam theory in which these assumptions were used are in good agreement with the experimental results, it is apparent that the only discernible effect of the adhesive was to add mass to the beams. Similar results were obtained in reference 8 for thin layers of adhesive.

Effects of Varying Air Density

For vibratory motion of a low-density beam in air, the mass of air being accelerated back and forth can add significantly to the moving mass, and lower the natural frequencies. Therefore, an experimental study was made of the effects of varying the density of the air surrounding a vibrating sandwich beam. Beam 2 was placed in a relatively large vacuum chamber and the natural frequencies corresponding to the first five modes were determined for various densities of the air surrounding the beam. The density of the air varied linearly with the pressure at constant temperature.

An analytical study of effects of varying the air density was also made for comparison with the experimental results. The analysis was based on the findings from the study of two-dimensional incompressible potential flow around a vibrating wing surface (as given in refs. 13 and 14, for example). The overall integrated effect of the air mass is as if a cylinder of air with a diameter equal to the wing chord is being carried back and forth with the wing. This air mass is usually referred to as the "apparent mass" or the "virtual mass." In the present analysis, each section of the beam is assumed to carry along with it an apparent mass of air contained in a cylindrical volume of air with a diameter equal to the beam width. It is to be recognized, however, that the beam amplitude, as well as surrounding airflow, does vary along the beam length and is, therefore, not two-dimensional. The analysis thus results in an approximation to the apparent-mass effect and also does not account for any effects of aerodynamic damping.

The weight of beam 2 was calculated by using the Timoshenko beam theory including secondary effects and was assumed to be composed of the apparent weight of the beam and the weight of the moving air surrounding the beam. The apparent weight of the beam for each mode was assumed to be constant and was determined by calculating the weight of the beam from the measured frequencies at atmospheric pressure and subtracting the weight of the air calculated from the assumed volume of air at the same pressure. The natural frequencies at various pressures were then calculated by using the Timoshenko theory with the weight used being the sum of the apparent weight of the beam as determined at atmospheric pressure and the calculated weight of the associated air.

The experimental and analytical natural frequencies corresponding to the first five modes of beam 2 are plotted against the pressure of the surrounding air in figure 4. For each mode, the experimental natural frequencies increased as the pressure, or density, decreased with a greater rate of change occurring at the lower pressures. (Had the near-vacuum value, rather than the atmospheric value, of each natural frequency been chosen as the reference value, each dashed-line theoretical curve would simply be displaced vertically and would pass through the near-vacuum value.) The experimentally determined frequency corresponding to the second mode had the maximum percentage change of 2.15 percent when the density of the surrounding air was reduced from a sea-level value to that of a near vacuum.

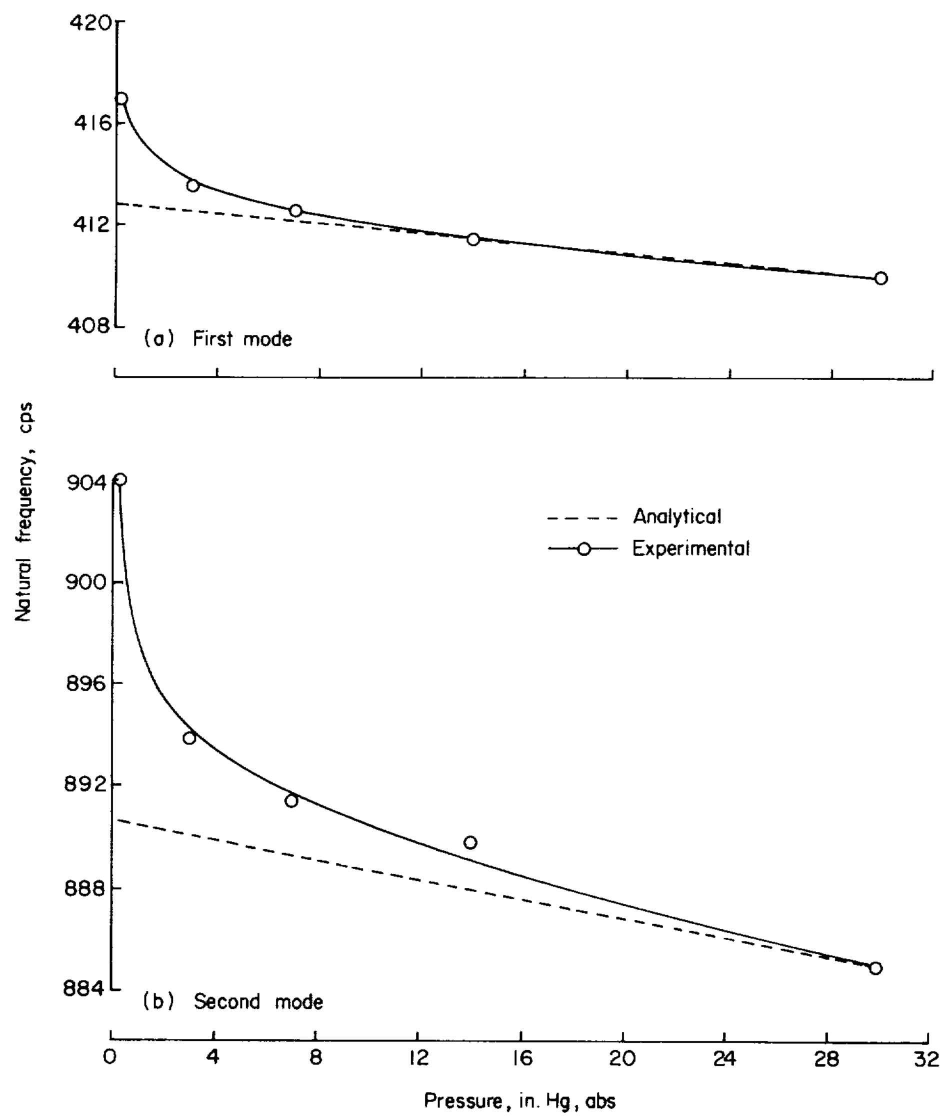


Figure 4.- Analytical and experimental variation with pressure of the natural frequency corresponding to the first five modes of beam 2.

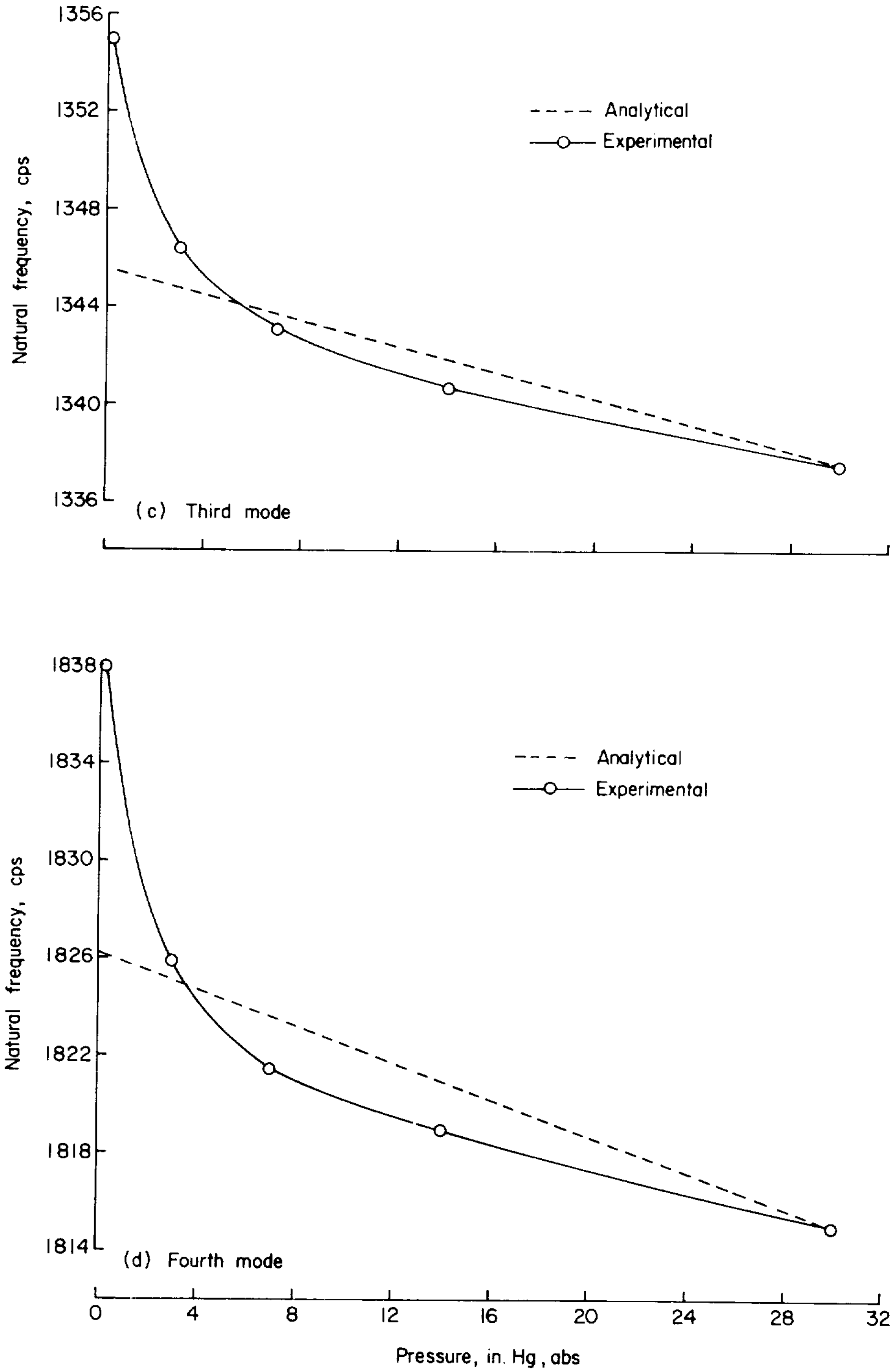


Figure 4.- Continued.

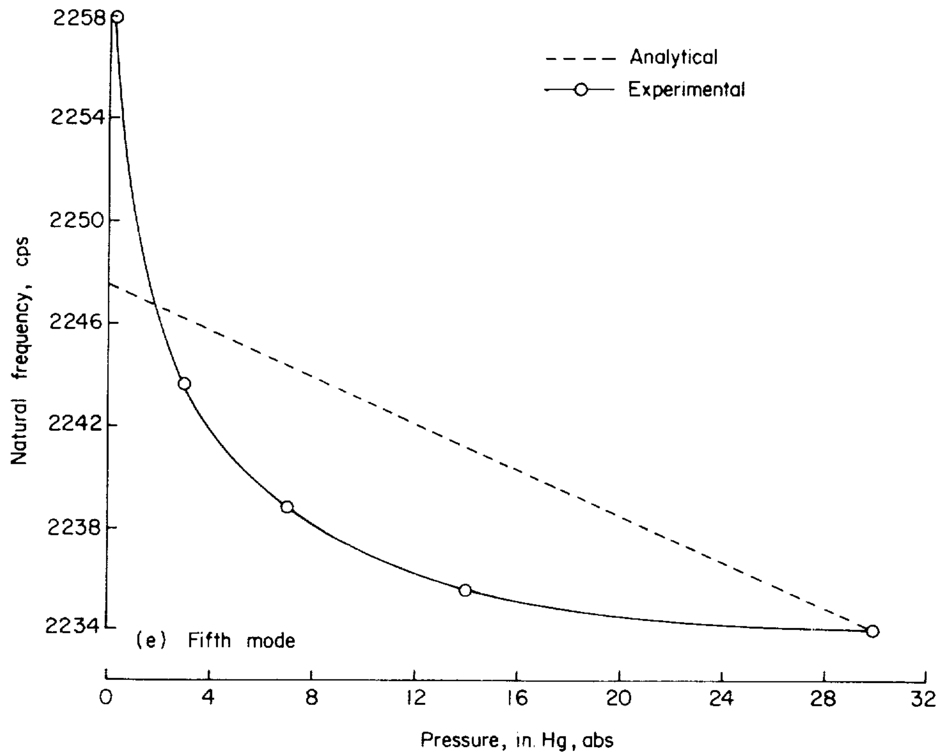


Figure 4.- Concluded.

It was apparent in the tuning of the natural frequencies at low pressures that the resonance peaks became very sharp and the point of maximum response of the beam was more easily determined. This observation indicated that there was less damping due to the air at the lower pressures.

These results show that when the density of the surrounding air medium is varied, there are small effects of apparent mass and of aerodynamic damping on the natural frequencies of a lightweight sandwich beam. The analytical method employed for comparison with experiment predicted an increase in the natural frequencies of a beam when the density of the surrounding air was decreased progressively to less than 1 percent of the sea-level value, but the analysis accounted for only about one-half of the experimental change in the frequencies. This lack of agreement may be due to the failure to account for the presence of the damping of the air, which would also reduce the measured resonant frequencies.

CONCLUDING REMARKS

The natural frequencies corresponding to the first six modes of each of three uniform rectangular-cross-section sandwich beams freely supported were determined by exciting each beam with an electromagnetic shaker. The natural frequencies corresponding to the first mode of each of the beams were also obtained by excitation with an air shaker.

Since the frequencies predicted by the classical beam theory were in poor agreement with the experimental results and the frequencies predicted by the Timoshenko beam theory were in good agreement with experimental results, it is evident that transverse shear and rotary inertia effects must be considered in the prediction of the natural frequencies of free-free sandwich beams. Although the secondary effect of rotary inertia was negligible for the beams tested, in general the rotary-inertia coefficient should be calculated to determine if this secondary effect can be safely neglected.

The flexural stiffness of the core of each beam was found to be negligible when compared with the flexural stiffness of the faces. The data indicate that the only effect of the thin layer of adhesive used in the bonding process was to add mass to the beams. Sandwich beams are normally very light and therefore the mass of any attachments to the beam should be added to the mass of the beam when predicting natural frequencies. The data indicate that increasing the face thickness of a sandwich beam can increase the natural frequencies of some modes, but the natural frequencies of higher modes may be reduced due to the influence of transverse shear.

Tests indicated that the natural frequencies of lightweight sandwich beams increase when the density of the surrounding air is decreased. The effect is small for the beam tested and can normally be neglected. The analytical method employed for comparison with experiment predicted an increase in the natural frequencies of a beam when the density of the surrounding air was decreased progressively to less than 1 percent of the sea-level value, but the analysis accounted for only about one-half of the experimental change in the frequencies. This lack of agreement may be due to the failure to account for the presence of the damping of the air, which would also reduce the measured resonant frequencies.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 3, 1963.

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